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USEFUL SCALING PARAMETERS FOR THE PULSE TUBE

J. M. Lee[1], P. Kittel[1], K. D. Timmerhaus[2], and R. Radebaugh[3]

[1]NASA Ames Research Center
Moffett Field, CA 94035-1000, USA

[2]University of Colorado, Boulder
Boulder, CO 80303, USA

[3]National Institute for Standards and Technology
Boulder, CO 80303, USA

ABSTRACT

A set of dimensionless scaling parameters for use in correlating performance data for Pulse Tube Refrigerators is presented. The dimensionless groups result after scaling the mass and energy conservation equations, and the equation of motion for an axisymmetric, two-dimensional ideal gas system. Allowed are viscous effects and conduction heat transfer between the gas and the tube wall. The scaling procedure results in reducing the original 23 dimensional variables to a set of 11 dimensionless scaling groups. Dimensional analysis is used to verify that the 11 dimensionless groups obtained is the minimum number needed to describe the system. We also examine 6 limiting cases which progressively reduce the number of dimensionless groups from 11 to 3. The physical interpretation of the parameters are described, and their usefulness is outlined for understanding how heat transfer and mass streaming affect ideal enthalpy flow.

INTRODUCTION

Experimentalists must often decide upon the most efficient way to correlate a large amount of laboratory data that will give a clear understanding of the results. This eventually leads to scaling the differential fluid equations, and then determining the number of mutually independent dimensionless groups using dimensional analysis..

As an example of the usefulness of dimensional analysis, consider heat transfer from a hot wire submerged in a moving fluid. The heat transferred per unit length of wire, *Q/l*, is a function of the temperature difference, □, wire diameter, *d*, fluid velocity, *u*; and the fluid density, □, heat capacity, C_p, viscosity □, and thermal conductivity, □. The number of data points required to measure these 8 variables a minimum of 3 times per variable is 3^8.

However, by using dimensional analysis, the above 8 variables can be reduced to 3 independent dimensionless groups

$$\frac{Q}{\lambda l \theta} = f\left(\frac{\rho u d}{\mu}, \frac{C_p \mu}{\lambda}\right),$$

which are the Nusselt, Reynolds and Prandtl Numbers where $Q = hA\theta$, with h as the heat transfer coefficient and $A = \pi d \cdot l$ as the heat transfer area. Thus only 3^3 independent measurements are required.

Limited progress has been made on modeling the Pulse Tube (PT) through solution of the fluid equations. Most models are based upon ideal one-dimensional (1D) flows[1]. To account for diffusion heat transfer and viscous effects, lumped-parameter corrections to the 1D models have been employed[2]. Direct solution of the two-dimensional (2D) fluid equations to account for heat transfer and viscosity have been made earlier in acoustics for boundary layer flows[3]. A recent 2D analysis has examined thermal and viscous diffusion and second order steady mass streaming for the case of negligible axial temperature gradient[4].

These models are not completely satisfactory. One-dimensional models cannot account for steady mass streaming, and solving for the temperature profile in 2D models is difficult. The limited progress is due to the complexity of the coupled differential equations for mass, momentum and energy conservation. Fortunately, some relief is available from dimensional analysis. Dimensional analysis gives the experimentalist the ability to correlate data without resorting to a complete solution of the differential equations. A recent dimensional analysis for thermoacoustics illustrates the usefulness of this approach[5].

The purpose of this paper is to present a formal dimensional analysis that suggests a set of useful dimensionless groups for correlating PT data. The dimensionless groups are shown to be consistent with the dimensionless groups obtained by scaling the 2D axisymmetric fluid equations. The analysis, being 2D, contains transverse heat transfer and viscous effects, which are the transport processes associated with departures from ideal 1D models. The set of dimensionless groups are then reduced for 6 special limiting cases of pulse tube operation.

SCALING

In this first section we scale the governing fluid equations that describe the flow dynamics of the tube. A sketch of the system is shown in Figure 1. Two problem domains are considered. The gas domain extends from $r^* = 0$ to $r^* = r_w^*$ and $z^* = 0$ to $z^* = L^*$ (starred variables are *dimensional* quantities). The tube wall domain extends from $y^* = 0$ to $y^* = l^*$ and $z^* = 0$ to $z^* = L^*$, where l^* is the tube wall thickness. Adiabatic conditions exist for the outer wall surface, and continuity of temperature and heat flux must exist between the gas and the tube wall interface. The velocity boundary conditions are of small amplitude and periodic so that time is described using complex notation: at $z^* = 0$, $u = U_0^* e^{i\omega^* t^*}$, and at $z^* = L^*$, $u = U_L^* e^{i(\omega^* t^* + \phi_U)}$ where $\omega^* = 2\pi f^*$ is the angular frequency, f^* is the frequency; t^* is time; ϕ_U is the velocity phase angle between the tube ends; and U_0^* and U_L^* are the velocity amplitudes at each end. The energy boundary condition at the outer tube wall is adiabatic, with the temperature at $z^* = 0$ and $z^* = L^*$ taken as $T^* = T_c^*$ and $T^* = T_h^*$, respectively. In this paper, we will refer to the z-direction as the axial direction, and the r-direction as the transverse direction. We will also use the following notation for partial derivatives: $\phi_{,\xi} = \partial\phi/\partial\xi$.

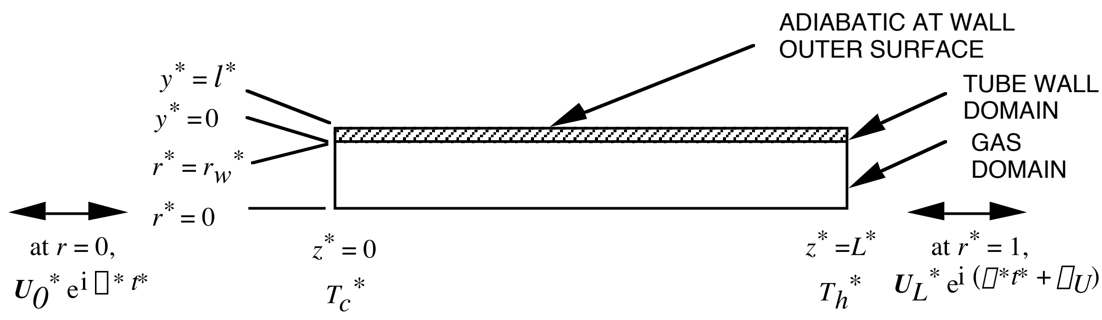

Figure 1. Axisymmetric system examined where $r_w^*/L^* \ll 1$.

The fluid equations[6] are reduced for our system using the following simplifying assumptions: axisymmetric cylindrical geometry; ideal gas; constant transport properties; Stokes' assumption for the second viscosity; and $r_w^{*2}/L^{*2} \ll 1$ (implying that $\partial p^*/\partial r^* \approx 0$ so that the r-momentum equation can be decoupled from the rest of the problem, and so that axial viscous transport is negligible in the z-momentum equation). The reduced fluid equations for mass conservation, equation of motion, energy conservation, and equation of state become, respectively,

$$\rho^*_{,t^*} + \frac{\left(\rho^* v^* r^*\right)_{,r^*}}{r^*} + \left(\rho^* u^*\right)_{,z^*} = 0\,;$$

$$\rho^*\left[u^*_{,t^*} + v^* u^*_{,r^*} + u^* u^*_{,z^*}\right] = -p^*_{,z^*} + \frac{\mu^*}{r^*}\left(r^* u^*_{,r^*}\right)_{,r^*}$$

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$$\rho^* C_p^*\left[T^*_{,t^*} + v^* T^*_{,r^*} + u^* T^*_{,z^*}\right] = \left(p^*_{,t^*} + u^* p^*_{,z^*}\right) + \frac{k^*}{r^*}\left(r^* T^*_{,r^*}\right)_{,r^*} + k^* T^*_{,z^*z^*} + \mu^* u^{*2}_{,r^*} \qquad (3)$$

$$p^* = \rho^* R^* T^*.$$

The energy conservation equation for the tube wall domain for $l^* \ll r_w^*$ is

$$\rho_t^* C_{pt}^* T^*_{,t^*} = k_t^*\left(T^*_{,y^*y^*} + T^*_{,z^*z^*}\right).$$

The time-averaged enthalpy flow is of primary interest since it represents refrigeration,

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$$\overline{H}^* = 2\pi\omega^* \oint_{1/\omega^*} \left[\int_0^{r_w^*} \rho^* u^* C_p^* T^* r^* dr^*\right] dt^*, \qquad (6)$$

where the overbar represents time-averaged values over a cycle. The kinematic velocity components in the z^* and r^* direction are u^* and v^*; the thermodynamic gas variables p^*, ρ^*, T^* are pressure, density and temperature, respectively; the density of the tube wall material is ρ_t^*; gas properties, μ^*, k^*, C_p^*, are the dynamic viscosity, thermal conductivity and heat capacity, and the tube wall properties, k_t^* and C_{pt}^*, are the thermal conductivity and heat capacity of the tube wall.

The above dimensional equations are next scaled (normalized) to dimensionless form so that the resulting dimensionless variables range from 0 to $\boldsymbol{O}(1)$ (order 1). The variables are scaled as follows: r^* is scaled with r_w^*, z^* is scaled with L^*, y^* is scaled with the tube wall thickness l^*, and t^* is scaled with the angular frequency ω^*; u^* is scaled with the axial boundary condition velocity U_0^*; v_0^* is scaled with ($U_0^* r_w^* / L^*$); p^*, ρ^*, T^*, are scaled with mean pressure p_0^*, reference density ρ_0^*, and reference temperature T_0^*. The transport properties μ^*, k^*, C_p^*, k_t^* and C_{pt}^*, and the tube wall density ρ_t^* are taken as constant. These scaling parameters are substituted into the dimensional Eqs. (1) to (6) and rearranged to give the corresponding dimensionless form (unstarred variables are *dimensionless*) for mass conservation, equation of motion, energy conservation, equation of state, tube wall energy conservation and time-average enthalpy flow, respectively:

$$\rho_{,t} + \sigma\left[\frac{(\rho v r)_{,r}}{r} + (\rho u)_{,z}\right] = 0 \qquad (7)$$

$$\rho\left[u_{,t} + \sigma\left(v u_{,r} + u u_{,z}\right)\right] = -\frac{\sigma}{\gamma \mathrm{M}^2} p_{,z} + \frac{1}{\mathrm{V}^2}\frac{\left(r u_{,r}\right)_{,r}}{r} \qquad (8)$$

$$\rho\left[T_{,t} + \sigma\left(v T_{,r} + u T_{,z}\right)\right] = \frac{\gamma - 1}{\gamma}\left(p_{,t} + \sigma u p_{,z}\right) + \frac{1}{\mathrm{P}^2\mathrm{V}^2}\left[\frac{\left(r T_{,r}\right)_{,r}}{r} + \frac{r_w^{*2}}{L^{*2}} T_{,zz}\right] + (\gamma - 1)\frac{\mathrm{M}^2}{\mathrm{V}^2} u_{,r}^2 \qquad (9)$$

$$p = \rho T \qquad (10)$$

$$T_{,t} = \frac{1}{\mathrm{F}^2}\left[T_{,yy} + \frac{l^{*2}}{L^{*2}} T_{,zz}\right]. \qquad (11)$$

$$\overline{H} = \pi r_w^2 \sigma \oint_{1/\sigma} \rho u T\,dt$$

EMBED "Equation" * mergeformat (12)

The enthalpy flow of the gas, Eq. (12), is normalized by the *sum of the axial heat conduction of the gas, the axial heat conduction of the tube wall, and the enthalpy flow of the gas at some reference point, all averaged over the transverse area* (which includes the gas and the tube wall). Since this quantity is constant in the axial direction when averaged over the transverse area (recall the adiabatic boundary conditions at the outer tube wall, Fig. 1), we take the reference enthalpy flow, $\overline{H}_{ref}^*$, to be at the cold end (this will be convenient for illustrating a later point in the section on dimensional analysis). The above set of equations identifies 9 dimensionless scaling groups. Two additional dimensionless groups, σ_U and U_L, enter through the boundary conditions. These groups are listed in Table 1 along with their physical meanings. The relative magnitudes of the groups provide an understanding of the importance of the various dynamic effects (friction, heat transfer, compressibility, etc.).

Table 1. Dimensionless Scaling Groups

	Name	**Definition**	**Physical Meaning**
ε	expansion parameter	$U_0^* / \left(\omega^* L^*\right) = d^* / L^*$	ratio of displacement length, d^*, to tube length
γ	heat capacity ratio	EMBED "Equation" * mergeformat C_p^* / C_V^*	ratio of constant pressure to constant volume heat capacities.
EMBED "Equation" * mergeformat V^2	Valensi Number	$r_w^{*2} \omega^* / \nu^*$	ratio of tube inner radius to viscous diffusion length
P^2	Prandtl Number	ν^* / α^*	ratio of viscous to thermal diffusion length scales
M	Mach Number	$U_0^* / \sqrt{\gamma R T_0^*}$	ratio of velocity at z = 0 to speed of sound
F^2	inverse Fourier Number	$\alpha_t^* / \left(\omega^* l_t^{*2}\right)$	ratio of thermal diffusion length to tube wall thickness
r_w^* / L^*	gas domain length ratio		ratio of tube radius to tube length
l^* / L^*	tube wall length ratio		ratio of tube wall thickness to tube length
$\overline{H}$	normalized enthalpy flow	$\overline{H}^* / \overline{H}_{ref}^*$	ratio of enthalpy flux to reference enthalpy flux $\overline{H}_{ref}^* = \rho_c^* T_c^* U_0^* C_p^* \pi r_w^{*2}$
U_L	velocity ratio	U_L^* / U_0^*	ratio of velocity amplitude at z = 1 to amplitude at and z = 0
θ_U	velocity phase angle		velocity phase angle at z = 1 relative to z = 0

DIMENSIONAL ANALYSIS

The Buckingham-Pi Theorem[7] states that the minimum number of dimensionless groups, Π, is determined from the expression

$$\Pi = m - n$$

where m is the minimum number of independent variables (such as velocity, pressure, thermal diffusivity, etc.) and n is the number of primary dimensions. For the pulse tube, we take time, length, mass and temperature to be our primary dimensions.

A helpful procedure used for obtaining the minimum number of independent variables (and therefore the minimum number of dimensionless groups) has been outlined by Krantz[8] and is summarized as follows:

1. List all equations necessary to solve for the quantity of interest.
2. List the boundary conditions.
3. Simplify the equations listed in steps 1 and 2 through use of any additional information. For example, neglecting higher order terms to reduce the number of variables, or using explicit solutions that can relate one variable in terms of others.
4. List all variables and group-of-variables that are mutually independent by examining the set of relations listed in 3. This typically requires finding natural groups within the equations and from the boundary conditions. The number of independent variables is "m"
5. List all fundamental dimensions (such as mass, length, time, temperature). The number of fundamental dimensions is "n".
6. The minimum number of independent dimensionless groups, Π, is m – n.
7. Combine the independent variables to form the minimum number of independent dimensionless groups.

Table 2. Full Complement of Dimensional Variables

Dimensional Variable	Symbol	Dimensional Variable	Symbol
enthalpy flow	$\overline{H}^*$	leading order pressure, gas	$p_0{}^*$
tube inner radius	r_w^*	leading order density, gas	$\rho_0{}^*$
tube length	L^*	leading order temperature, gas	$T_0{}^*$
tube thickness	l^*	oscillating pressure, gas	$p_1{}^*$
angular frequency	ω^*	oscillating density, gas	$\rho_1{}^*$
oscillating radial velocity	v_0^*	oscillating temperature, gas	T_1^*
oscillating axial velocity	u_0^*	dynamic viscosity	μ^*
axial velocity at z=0	U_0^*	thermal conductivity, gas	k^*
axial velocity ratio	U_L^*	heat capacity, gas	C_p^*
axial phase angle	θ_U	density, tube	$\rho_t{}^*$
mass ideal gas constant	R	thermal conductivity, tube	$k_t{}^*$
		heat capacity, tube	$C_{pt}{}^*$

In keeping with the methodology outlined above, the first step in dimensional analysis is to identify the quantity of interest. This is the time and (transverse) area averaged enthalpy flow, $\overline{H}^*$, with C_p^* taken as constant. In general, it is a nonlinear quantity, the time-averaged product between mass flux and temperature (Eq. 6) or velocity and pressure,

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$$\overline{H}^* = 2\pi\omega^* \frac{\gamma}{\gamma-1} \int_0^{r_w^*} \left[\int_0^{2\pi/\omega^*} p^* u^* dt^* \right] r^* dr^* . \qquad (13)$$

Determining $\overline{H}^*$ requires solving for p^* and u^* over the domain. This generally requires solving the full 3D set of fluid equations. However, we can immediately reduce the equations to 2D axisymmetric as given by Eqs. (1) to (5).

The next step is to identify the boundary conditions. The velocity can be of different amplitude (U_0^* and U_L^*) and phase (θ_U) as indicated in Figure 1. The temperature (energy) boundary condition at the outside of the tube is adiabatic. Temperature and heat flux must be continuous across the gas/tube interface.

The equation set can be further simplified by linearizing if the parameter ε is small (see Eqs (7), (8) and (9)). An asymptotic series solution in the small parameter ε with M << ε can then be used to obtain an anelastic linear set of leading order equations with second order corrections. Details of this expansion are given elsewhere.[9]

The full complement of 23 dimensional variables relevant to the problem is listed in Table 2. These are not all mutually independent. The Pi Theorem requires the minimum number of variables, that is, the number of variables that are mutually independent. The independent variables are obtained by considering additional information. In particular, the number of variables listed in Table 2 can be reduced by finding "natural groups" within the equations, by using information from the boundary conditions, and by using explicit equations relating variables. The following discussion makes use of this additional information.

Thermal Diffusivity for the Tube Wall. The energy equation for the tube wall is simply the unsteady diffusion equation, with the thermal diffusivity, $k_t^* / \left(\rho_t^* C_{pt}^* \right)$ being a natural group that describes the relative importance between unsteady effects and diffusion. The result of defining this natural group is to combine 3 variables and reduce them to a single independent group. Thus the number of variables is reduced by 2.

Ideal Gas Relation. From linearization, The zeroth and first order dependence of pressure, density and temperature is described by the corresponding zeroth and first order equation of state for an ideal gas. This constitutes 6 unknowns (zeroth order: p_0^*, ρ_0^*, T_0^*, and first order: p_1^*, ρ_1^*, T_1^*) in two equations ($p_0^* = \rho_0^* T_0^*$, $p_1^* = \rho_0^* T_1^* + \rho_1^* T_0^*$), hence there are 4 degrees of freedom and thus the number of variables is reduced by 2 (= 6 – 4).

Adiabatic Boundary Conditions. The adiabatic boundary conditions at the outer tube wall requires that the gas enthalpy flow *plus* conduction for the total (gas and tube wall) system to be constant. This relation reduces the number of variables by 1.

Analytic Solution for u_0^*. An analytic solution for u_0^* in z^* is obtained directly from the linear momentum equation[9]

EMBED "Equation" * mergeformat $$u_0^* \quad \left[\left(U_L^* e^{i\phi_U} - U_0^*\right) z^* + U_0^*\right] F\left(r^*\right)$$

where $F(r^*)$ is the functional radial dependence and represents the real part. Thus u_0^* (which is the quantity of interest to us) contains the effects of three variables, U_0^*, U_L^* and ϕ_U. The number of variables is reduced by 3.

The total number of variables has been reduced from the original 23 to 15, hence m = 15. We now take time, length, mass and temperature to be our fundamental dimensions, so that n = 4. Finally we arrive at the minimum number of independent dimensionless groups, which is 11.

The set of variables are now formed into 11 dimensionless groups. With the help of the previous scaling discussion, we choose the dimensionless groups listed in Table 1. These groups are mutually independent, hence, these are the parameters to vary when conducting experiments (all other things being equal, i.e., regenerator, heat exchanger and compressor performance being constant). The 11 groups, in conjunction with experimental data, quantify how heat transfer and secondary mass streaming influence the enthalpy flow and the temperature difference between the tube ends.

LIMITING CASES

This dimensional analysis has resulted in reducing the original 23 variables to 11 independent dimensionless groups. The 11 dimensionless groups listed in Table 1 are reiterated in Table 3 as Case 0. It is the most complex case for correlating data based on our reduced problem (Eqs. (1) to (6) and the boundary conditions of Fig. 1). Mapping the entire parameter space for these 11 groups is still a formidable task. Fortunately, there are several limiting cases for the PT that further reduce the number of dimensionless groups. This is accomplished by expanding for small values of a particular dimensionless group, which implies negligible effects for that group (at leading order). We present 6 cases.

Case 1: Small Mach number limit and constant γ. The pulse tube operates at the small Mach number limit (typically, M = $\boldsymbol{O}(10^{-3})$). This reduces the number of dimensionless groups from 11 to 10 and effectively reduces the problem to an anelastic one with negligible (energy) viscous dissipation. In addition, for practical purposes, we can simply take γ as large as possible, since a larger γ results in larger temperature oscillations (for an isentropic process), hence larger enthalpy flows. Helium, a monatomic molecule

Table 3. Suggested Dimensionless Groups for Various Limiting Cases

Case 0	**Case 1**	**Case 2**	**Case 3**	**Case 4**	**Case 5**	**Case 6**
full 2D	M<<1 and γ = constant	Case 1 plus isothermal wall	Case 1 plus adiabatic wall	Case 3 plus no viscous flow	Case 4 plus $T_{,z}$ = constant	Case 5 plus □ = 90°
M						
γ						
l^{*2}/L^{*2}	l^{*2}/L^{*2}					
F^2	F^2					
P^2	P^2	P^2				
V^2	V^2	V^2	V^2			
r_w^{*2}/L^{*2}	r_w^{*2}/L^{*2}	r_w^{*2}/L^{*2}	$r_w^{*2}/L^{*2}P^2V^2$	$r_w^{*2}/L^{*2}P^2V^2$		
□U	□U	□U	□U	□U	□U	□U = 90°
□ U_L	□ U_L	□ U_L	□ U_L	□ U_L	□ U_L	□ U_L
$\overline{H}$	$\overline{H}$	$\overline{H}$	$\overline{H}$	$\overline{H}$	$\overline{H}$	$\overline{H}$

whose γ is 5/3, is usually used as the working fluid. Constant γ further reduces the number of groups from 10 to 9.

Case 2: Isothermal wall. The isothermal tube wall limit implies that the thermal inertia of the wall is infinite, i.e., the temperature of the gas is pinned at the wall. This eliminates the effect of the wall domain, groups (l^{*2}/L^{*2}) and (F^2), and so reduces the number of groups from 9 to 7.

Case 3: Adiabatic wall. The adiabatic limit for the tube wall implies that the tube wall has no effect on the temperature oscillations of the gas (negligible heat capacity of the wall). The adiabatic boundary condition at the outer tube wall is directly seen by the gas, and is characterized by the limit in which the dimensionless groups for the tube wall domain go to zero, (l^{*2}/L^{*2}) $\to$ 0 and ($l^{*2}/L^{*2}F^2$) $\to$ 0. At this limit, there is no heat flux at the gas/wall interface, which effectively eliminates the tube wall domain. There is also no heat flux at the centerline due to symmetry. These temperature boundary conditions eliminate transverse thermal diffusion thus requiring the temperature to be constant in the radial direction (see Eq. (9)). All this leads to a reduction of the number of dimensionless groups from 9 to 6. Axial heat conduction remains and is now characterized by the group $r_w^{*2}/\left(L^{*2}P^2V^2\right)$.

Case 4: Adiabatic wall with no viscosity. This is the same as Case 3 but with the additional constraint of $1/V^2 \to 0$. This further reduces the number of groups from 6 to 5 and effectively eliminates all transverse diffusion effects, hence, the problem becomes one-dimensional.

Case 5: Adiabatic wall with no viscosity and constant axial temperature gradient of the gas. This is the same as Case 4 with the additional assumption of a constant axial temperature gradient (axial linear temperature profile). A constant temperature gradient eliminates the effect of axial heat conduction scaling group $r_w^{*2}/\left(L^{*2}P^2V^2\right)$(see Eq. 9). In real systems, though, this may not be the case.

Case 6: Adiabatic wall with no viscosity, constant axial temperature gradient of the gas, and ideal integrated flow through the orifice. This is the same as Case 5 with the additional requirement that the phase angle between the velocities at the two ends is 90° (or equivalently, the phase angle between pressure in the tube and mass flow through the orifice is 0°). This is usually the case in ideal models, however, it is not necessarily the case in which there is active control of the hot end mass flow, such as for moving plug systems or systems in which the orifice is replaced with solenoid valves.

As an example of how these dimensionless groups may be used, consider the simplest case of ideal enthalpy flow, Case 6 in Table 3. Case 6 contains 3 independent dimensionless parameters, □ U_L, and $\overline{H}$. These parameters represent the following measurable quantities: □ represents the dynamic pressure oscillation relative to the mean pressure, or equivalently, the pressure ratio (□ can be written as d^*/L^* where d^* is the (imaginary) piston displacement at one end of the tube and L^* is the tube length); $U_L = U_L^*/U_0^*$ represents the mass flow ratio between the two ends; and $\overline{H}$ is the normalized enthalpy flow which is determined by the amount of heat rejected at T_h for a given T_c. Measurements of the dimensional data (U_0^* may be a bit difficult), for the prescribed conditions of $\{l^{*2}/L^{*2}\}$, $\{l^{*2}/(L^{*2}F^2)\}$ and {1/¥^{2}} being small, and the axial temperature profile being reasonably linear (which may not necessarily be the case) will allow a correlation between the three groups. These groups are also indirectly suggested by Storch and Radebaugh, et. al.[10] (in their Eqs. (2-39) and (2-40)) who give a 3 parameter functional relation that can be rewritten in terms of □ U_L, and $\overline{H}$.

CONCLUSION

A set of 11 dimensionless scaling groups have been identified for use in correlating pulse tube data. The 11 groups allow for transverse heat transfer to the tube wall and viscous effects. For ideal one-dimensional enthalpy flow in which the phase angle between the pressure in the tube and mass flow through the orifice is in phase, and the axial temperature is reasonably linear, the number of dimensionless groups reduce to 3: pressure ratio, velocity ratio between the two ends, and normalized heat rejected at the hot end.

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$$\frac{\dot{m}_h^* T_h^*}{\dot{m}_c^* T_c^*} = \frac{(\square_h^* U_L^* A) T_h^*}{(\square_c^* U_0^* A) T_c^*} = \frac{U_L^*}{U_0^*} \frac{(\square_h^* T_h^*)}{(\square_c^* T_c^*)} = \frac{U_L^*}{U_0^*} \frac{p_0^*/R}{p_0^*/R} = U_L.$$